Part II

Logarithmic Equations--Equation with one or more logs

ex
$\log_{100} x = -3/2$

$100^{-\frac{3}{2}} = x$

$\frac{1}{100^{\frac{3}{2}}} = x$

$\frac{1}{\sqrt{100}^3} = x$

ex
$\log_x 64 = 3$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

ex
$\log_x \sqrt{5} = 3/2$

$(x^{\frac{1}{2}})^{\frac{2}{3}} = (\sqrt{5})^{\frac{2}{3}}$

$x = (5^{\frac{1}{2}})^{\frac{2}{3}}$

$x = 5^{\frac{1}{3}}$

$x = \sqrt[3]{5}$

ex
$\log_x 3 = -2$

$(x^{-2})^{-\frac{1}{2}} = 3^{-\frac{1}{2}}$

$x^1 = \frac{1}{3^{\frac{1}{2}}} = \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

ex
$\log_{4x} 9 = 2$

$(4x)^2 = 9$

$16x^2 = 9$

$\sqrt{x^2} = \sqrt{\frac{9}{16}}$

$x = \pm\frac{3}{4}$ $\quad +\frac{3}{4}$

Do:

$(x^{\frac{3}{4}})^{\frac{4}{3}} = 27^{\frac{4}{3}}$

$x = 81$

1. $\log_x 27 = 3/4$
2. $\log_6 x = 3$ $\quad 6^3 = x$, $216 = x$
3. $\log_x 7 = 1/2$
4. $\log_x \sqrt{5} = 1/4$

Inequalities

If $b > 1$, $x > 0$

and $\log_b x > y$, then $b^y < x$

and $\log_b x < y$, then $b^y > x > 0$

$0 < x < b^y$

Ex
$\log_8 x < 2$

$8^2 > x$

$0 < x < 64$

Ex
$\log_5 x > 3$

$5^3 < x$

$x > 125$

$\log_b x = \log_b y$ iff $x = y$

ex
$\log_3 (x + 2) = \log_3 (2x)$

$x + 2 = 2x$

$2 = x$ ✓

ex

$\log_{10}(3x - 4) < \log_{10}(x + 6)$

$3x - 4 < x + 6$

$2x < 10$

$\frac{4}{3} < x < 5$

Restrictions

$3x - 4 > 0$

$x > 4/3$

$x + 6 > 0$

$x > -6$

Cannot take the log of negative or zero

ex

$\log_4 x^2 = \log_4 (4x - 3)$

$x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3 \quad x = 1$

$\{1, 3\}$ ✓

HW
p536
47-61odd, 54, 62